AS91524: Demonstrate understanding of electrical systems

Level 3 Credits 6

This achievement standard involves demonstrating understanding of electrical systems.

Achievement	Achievement with Merit	Achievement with Excellence
Demonstrate understanding of electrical systems.	Demonstrate in-depth understanding of electrical systems.	Demonstrate comprehensive understanding of electrical systems.

Assessment is limited to a selection from the following:

Resistors in DC Circuits
Internal resistance; simple application of Kirchhoff's Laws.

Capacitors in DC Circuits
Parallel plate capacitor; capacitance; dielectrics; series and parallel capacitors; charge/time, voltage/time and current/time graphs for a capacitor; time constant; energy stored in a capacitor.

Inductors in DC Circuits
Magnetic flux; magnetic flux density; Faraday's Law; Lenz's Law; the inductor; voltage/time and current/time graphs for an inductor; time constant; self inductance; energy stored in an inductor; the transformer.

AC Circuits
The comparison of the energy dissipation in a resistor carrying direct current and alternating current; peak and rms voltage and current; voltage and current and their phase relationship in LR and CR series circuits; phasor diagrams; reactance and impedance and their frequency dependence in a series circuit; resonance in LCR circuits.

Relationships:

$E = \frac{1}{2}QV$ $\quad Q = CV \quad C = \frac{\varepsilon_o \varepsilon_r A}{d} \quad C_T = C_1 + C_2 + \ \dots \quad \tau = RC$

$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \dots \quad \phi = BA \quad \varepsilon = -L\frac{\Delta I}{\Delta t} \quad \varepsilon = -\frac{\Delta \phi}{\Delta t}$

$\frac{N_p}{N_s} = \frac{V_p}{V_s} \quad E = \frac{1}{2}LI^2 \quad \tau = \frac{L}{R}$

$I = I_{MAX} \sin \omega t \quad V = V_{MAX} \sin \omega t \quad I_{MAX} = \sqrt{2} I_{rms}$

$V_{MAX} = \sqrt{2} V_{rms} \quad X_C = \frac{1}{\omega C}$

$X_L = \omega L \quad V = IZ \quad \omega = 2\pi f \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$

This achievement standard replaced unit standard 6389, unit standard 6390, and AS90523.

Basic Electricity

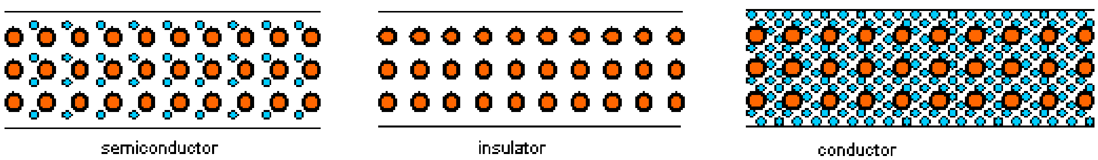
Stuff you should know from Level 2 (or lower)

Materials that will conduct electricity are called electrical **CONDUCTORS**. Those that won't are called **INSULATORS**.

Solids that will conduct electricity (by using batteries alone): all metals (they contain a lot of free electrons) and carbon

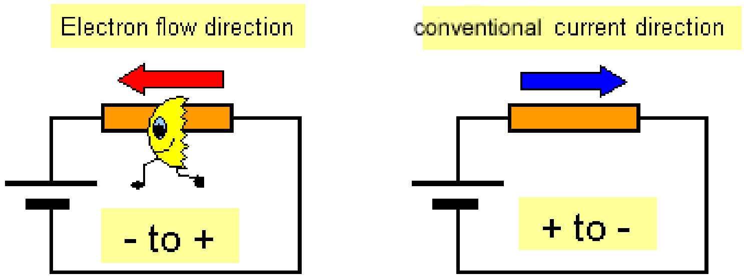
Materials that have a very high resistance are called **INSULATORS** and those that have a low resistance are called **CONDUCTORS**.
A piece of wire is made of millions of atoms and each one of these has its own cloud of electrons. However in a metal there is a large number of electrons that are not held around particular nuclei but are free to move at high speed and in a random way through the metal. These are known are **free electrons** and in a metal there are always large numbers of these. It is when these free electrons are all made to move in a certain direction by the application of a voltage across the metal that we have an electric current.

The difference between a metal (a large and constant number of free electrons), a semiconductor (a few free electrons, the number of which varies with temperature) and an insulator (which has no free electrons):



Charge and current

As a charge moves round a circuit from the positive to the negative it loses energy. There is a problem here. As you know an electric current is a flow of negatively charged electrons and these flow away from the negative terminal of a supply, round the circuit and back to the positive terminal. However the "conventional" view of current flow is from positive to negative and we will take that view when looking at the energy of electrical charge.



Each electron has only a very small amount of electric charge, and it is more convenient to use a larger unit when measuring practical units of charge. This unit is the **coulomb**. The charge on one electron is -1.6×10^{-19} C(usually written as *e*) You would need about 5×10^{18} electrons to have a charge of one coulomb.

Electric current is the rate of flow of charge round a circuit. The current at a point in the circuit is the amount of charge that passes that point in one second.

Electric current is measured in **AMPERES** (AMPS, symbol A).

(Charge must be given in coulombs, current in amps and time in seconds)

Current (I) = Charge (Q) / Time (t)
or
Charge (Q) = Current (I) x Time (t)

1 A = 1000 milliamp (mA)
1 A = 1 000 000 microamps (μA)

An alternative definition of the ampere (amp) based on fundamental quantities is:

A current of one amp is flowing in two parallel conductors placed one metre apart in a vacuum when there is a force between them of $2x10^{-7}$ Nm^{-1}.

Electrical Circuits

When one, or more, electrical components are joined together to a cell it is called a circuit. Electricity will not flow if there are any breaks in this circuit.

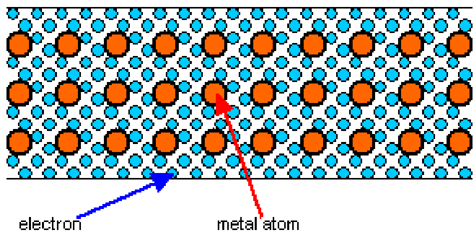
Current and Voltage

Current is the rate of flow of charge through a wire.
Voltage is a measure of the energy of that charge.

Energy is measured in joules and so we need to know the connection between volts and joules.

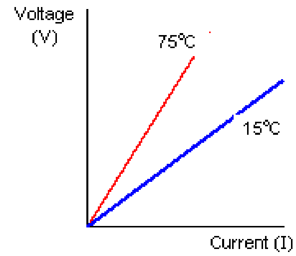
The potential difference (p.d.) between two points in a circuit is 1 V if 1 joule of electrical energy is changed to other forms of energy when 1 C passes from one point to the other.

Resistance



The free electrons in a metal are in constant random motion. If a potential difference is now applied across the metal the electrons tend to move towards the positive connection. As they do so their progress is interrupted by collisions. These collisions impede their movement and this property of the material is called its resistance. If the temperature of the metal is raised the atoms vibrate more strongly and the electrons make more violent collisions with them and so the resistance of the metal increase.

The resistance of any conducting material depends on the following factors:



- the material itself (actually how many free electrons there are per metre cubed)
- its length
- its cross-sectional area
- its temperature

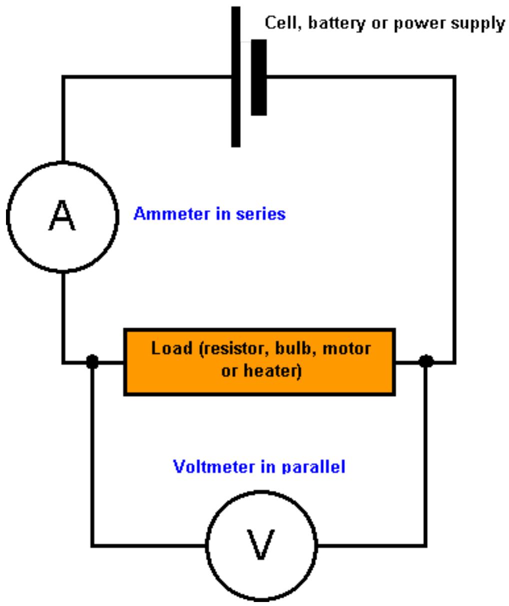
Series and Parallel Circuits

If you have a battery and two bulbs they can be connected in series or parallel.

If the batteries and bulbs in both circuits are the same then:
(a) the bulbs in the parallel circuit will be brighter than those in the series circuit
(b) the battery in the parallel circuit will run down quicker than the one in the series circuit

Connecting an ammeter and a voltmeter

The diagram shows the correct connections for an ammeter and a voltmeter. The ammeter is connected in series with the load and the voltmeter is connected in parallel with the load.



Ohm's Law

This result was first discovered by a man named Ohm and so it is called Ohm's Law.

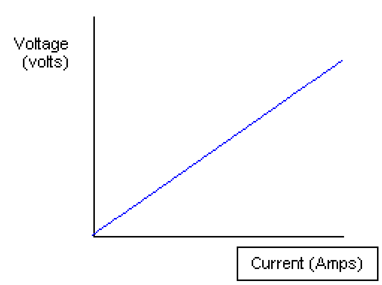
Ohm's Law states that:

> The ratio of the current in a conductor to the potential difference (voltage difference) between it ends is a constant as long as the temperature stays constant.
> This constant is called the RESISTANCE of the conductor.

You can write this in an equation as:

> Resistance = Voltage (V)/Current(I) or R = V/I

Resistance is measured in units called Ohms (Ω).If you plot a graph of current through a piece of wire against the voltage applied you should get a result like the one shown in the diagram below (the temperature of the wire must not change).



Resistance and heating

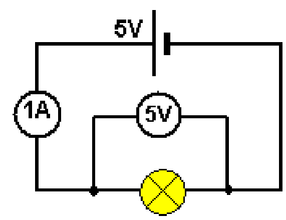
When an electric current flows through a wire the wire heats up. This is because the electrons collide with the atoms of the metal as they move and the atoms absorb some of their energy. This makes the atoms vibrate more strongly and so the wire heats up.

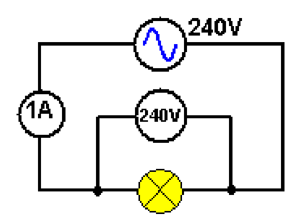
The bigger the current in a wire the bigger the heating effect

The filament in an electric light bulb is very thin and has a high resistance. The heating effect of the electric current flowing through the filament is so great (the temperature of the filament may reach over 1500^{o}C) that the wire glows – this is how electrical energy is converted to light energy.

It is possible to have two wires carrying the same current but one with much more energy than the other.

The low voltage bulb has a current of 1 A flowing through it, it runs on 5 V and gives out energy as heat and light. However the mains bulb runs on 240 V and also has a current of 1 A flowing through it. It gives out a lot more energy and so is much brighter.



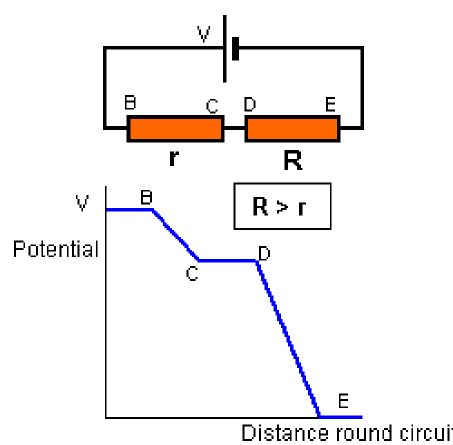


Potential and Potential Difference

We define the amount of electrical potential energy that a unit charge has as:

The electrical potential energy of a unit charge at a point in a circuit is called the potential at that point.

The diagram shows how the potential varies round a basic circuit. (The energy lost in the connecting wires is negligible). This means that the energy of the charge at one end of a connecting wire is the same as that at the other end. The bigger the energy change the bigger the difference in potential. We call the difference in electrical potential between two points in the circuit the potential difference between those two places.



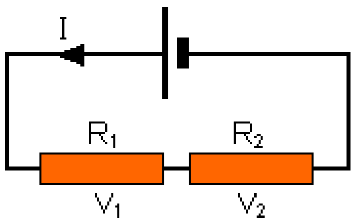
The potential difference between two points is defined as:

Potential difference between two points in a circuit is the work done in moving unit charge (i.e. one coulomb) from one point to the other

The units for potential difference are therefore Joules per coulomb, or volts.
(1 volt = 1 Joule/coulomb).

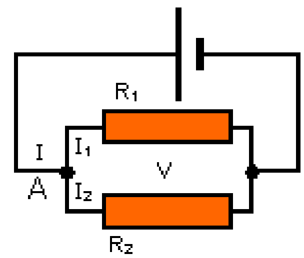
Series and parallel resistors

Two resistors in series



Resistors in series: $R = R_1 + R_2$

Two resistors in parallel

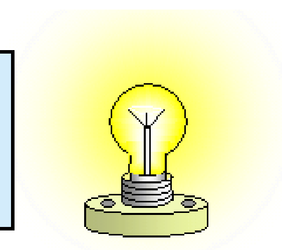


Resistors in parallel: $1/R = 1/R_1 + 1/R_2$

Two resistors in *series* always have a *larger* effective resistance than either of the two resistors on their own, while two in *parallel* always have a *lower* resistance.

Electrical Power

A light bulb may be marked 100W
A normal one bar electric fire is usually 1 kW
A TV may be rated at 250 W
A small electric motor may run at 5 W
An immersion heater may have a power of 3 kW
An electric clock may have a power of 1 mW



These figures tell you how **POWERFUL** the appliance is (how much electrical energy it uses in a second).

In one second the 100W light bulb uses 100 J of electrical energy and converts some of it into light energy.

In one second the immersion heater uses 3 kW (3000 W) of electrical energy and converts it into heat energy.

Electrical power = voltage x current

Power = VI

For large amounts of power we use kilowatts kW (1 kW = 1000 W) and megawatts (1 MW = 1000 000 W).

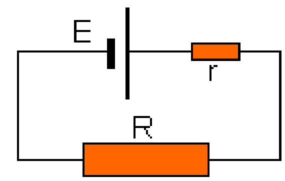
Using Ohm's law (V = IR) we can derive two alternative versions of the power equation:

$$\text{ELECTRICAL POWER} = VI = I^2R = V^2/R$$

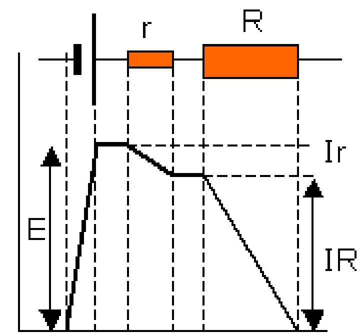
Electromotive force and internal resistance

When current flows round a circuit energy is transformed in both the external resistor but also in the cell itself. All cells have a resistance of their own and we call this the **internal resistance** of the cell.

The voltage produced by the cell is called the **electromotive force** or E.M.F for short and this produces a p.d across the cell and across the external resistor.



The E.M.F of the cell can be defined as the maximum p.d that the cell can produce across its terminals, or the open circuit p.d since when no current flows from the cell no electrical energy can be lost within it.



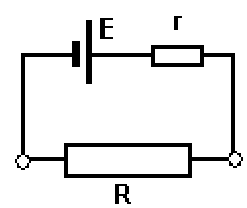
If the E.M.F of the cell is E and the internal resistance is r and the cell is connected to an external resistance R then:

$$E = V + Ir = IR + Ir$$

The quantity of useful electrical energy available outside the cell is *IR* and *Ir* is the energy transformed to other forms within the cell itself.

We usually require the internal resistance of a cell to be small to reduce the energy transformed within the cell; however it is sometimes helpful to have a rather larger internal resistance to prevent large currents from flowing if the cell terminals are shorted.

Maximum power transfer theorem



The external resistance will affect the current drawn from a source of E.M.F, and therefore the energy lost within it since the same current will also flow through the internal resistance. It is possible to find the value of this external resistance R that will give the greatest power output.

E = IR + Ir

r = R for maximum power output and so the resistance of the load should be equal to the internal resistance of the supply.

This is the case for an amplifier and loudspeaker; the output impedance of the amplifier should be matched to that of the speaker. In other words if the output impedance of the amplifier is 15Ω the resistance of the speaker should also be 15Ω.

(However this condition is not necessarily the most efficient operating state of the system)

Kirchoff's Rules

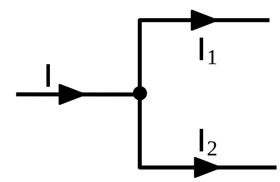
These two important rules apply to all electrical circuits and are particularly helpful when dealing with branched circuits:

1. The algebraic sum of the currents at a junction is zero. In other words there is no build up of charge at a junction

2. The sum of the changes in potential round a closed circuit must be zero.

Rule 1 is about charge conservation while rule 2 is about energy conservation.

We can see how these rules work by considering the circuit shown in Figure 1.



Rule 1
At the point B there is a junction
Current flowing from the cell (I) = Current in R_1 (I_1) + current in R_2 (I_2)

Rule 2
Round loop A,B,C,D,E: p.d across cell = - p.d across R_1
This represents a gain of potential in the cell but a loss in R_1

Round loop B,C,D,B: p.d across R_1 = - pd across R_2
In this equation there is a minus because we are moving 'against' the current in R_2

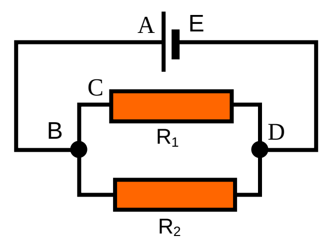


Figure 1

Example problem
Consider the circuit shown in Figure 2.
Calculate the current shown in each resistor.

The directions of current have been chosen arbitrarily.
When we have finished the calculation it may be that we have chosen them incorrectly.
If so we can simply reverse them on the diagram.

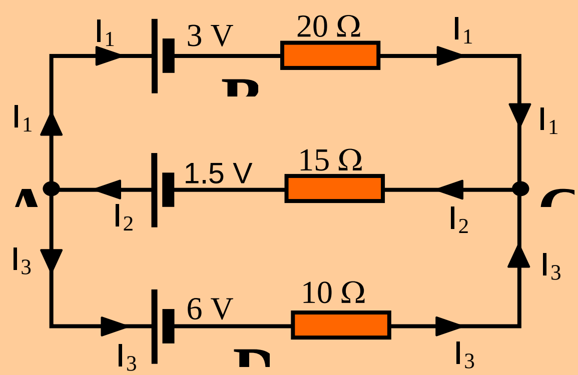


Figure 2

(a) Rule 1
Apply rule 1 to junction A
$I_2 = I_1 + I_3$ (1)

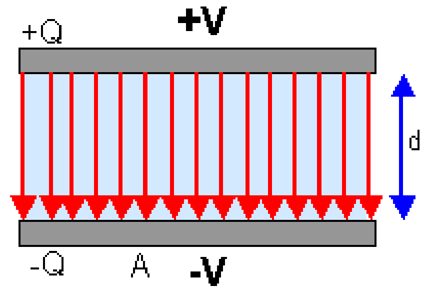
(b) Rule 2
(i) Choose the loop ABC
The sum of the e.m.f.s round this loop is the sum of the (IR) products in the loop.
$1.5 - 3 = 20\ I_1 + 15\ I_2$
Therefore: $-1.5 = 20\ I_1 + 15\ I_2$ (2)

(ii) Now consider the loop ADC
The sum of the e.m.f.s round this loop is the sum of the (IR) products in the loop.
$1.5 - 6 = 10\ I_3 + 15\ I_2$
Therefore: $-4.5 = 10\ I_3 + 15\ I_2$ (3)

Using equations 1 and 2: $-1.5 = 20\ I_2 - 20\ I_3 + 15\ I_2 = 35\ I_2 - 20\ I_3$ (4)
Now using equations 3 and 4: $-9 = 20\ I_3 + 30\ I_2$
Solving these gives: $I_1 = 0.046$ A, $I_2 = -0.162$ A, $I_3 = -0.208$ A

Capacitors (in D.C. circuits)

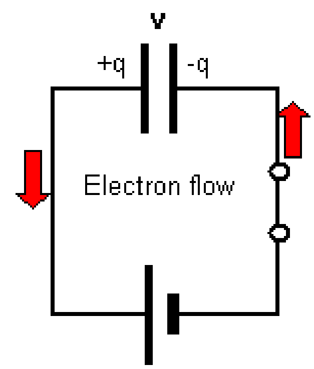
The parallel-plate capacitor



In its most basic form a capacitor is simply two metal plates with a material of permittivity ε filling the space between them. Such an arrangement is called a parallel plate capacitor. The plates of a charged parallel capacitor each carry charges of the same size but of opposite sign.

The electric field strength between the plates can be calculated using:

$$E = V/d$$



Electrons build up on one plate (-V) and move away from the other plate (+V).

A capacitor builds up charge but the charges never actually move between the plates (the spark would damage the capacitor).

The basic capacitor formula is:

Capacitance (C) = εA/d

Therefore the capacitance increases if the area of the plates is increased or their separation decreased.

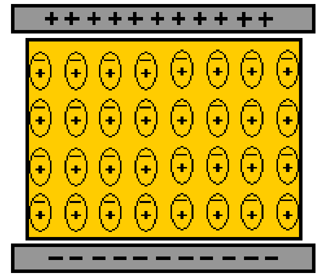
The insertion of a material with a high permittivity will increase the capacitance of a capacitor.

An air filled parallel plate capacitor with plates 1 mm apart and with a capacitance of 1 F would have plates each of a tenth of a square kilometre!

$C = Q/V$

Capacitors vary greatly in size but most are very small. If you examine different capacitors, you will find the capacitance, usually in μF, marked on them.
1 μF = 10^{-6} F
1 pF = 10^{-12} F

The action of a dielectric



When a dielectric material is placed between the plates of a parallel-plate (or other) capacitor the capacitance increases.

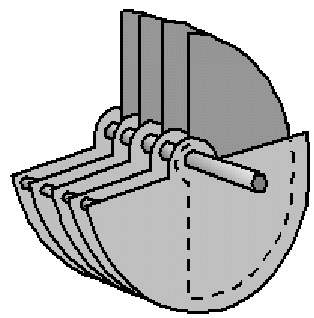
The charges on the plates of the capacitor induce opposite charges on the two surfaces of the dielectric. This has the effect of reducing the potential difference across the capacitor.
The capacitance of a parallel-plate capacitor with a material of relative permittivity ε_r filling the space between the plates is

$$\text{Capacitance (C)} = \varepsilon_0\varepsilon_r A/d$$

where ε_r is the ratio of the capacitances when the space between the plates is a vacuum or a dielectric.

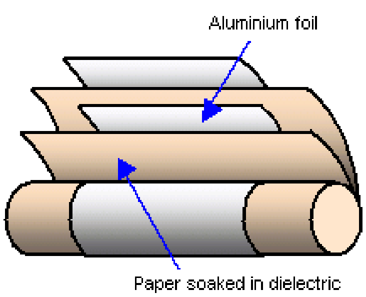
Practical forms of capacitor

Most capacitors are of the parallel-plate type, either as two flat plates or as a 'swiss roll' arrangement, with the plates and dielectric rolled into a cylinder.



The *air capacitor* has the advantage of being simple to make and having a precisely known capacitance with almost perfect properties at all frequencies.
The tuner in a radio is a variable air capacitor, consisting of two sets or plates in air overlapping each other. The overlap of the plates and hence the capacitance may be varied by moving one set of plates into the other.

Paper capacitors are thin metal sheets, with a paper dielectric of relative permittivity of about 5 between them. They are then rolled into a cylinder. The whole arrangement is packed in a cylinder of metal or plastic. Such capacitors are not very stable but they are cheap to make.

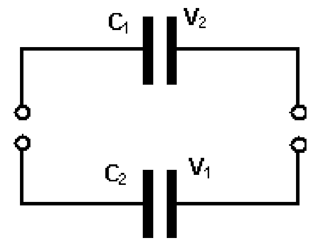


Electrolytic capacitors are made by passing an electric current through a solution of aluminium borate using aluminium electrodes. When this happens a very thin layer of oxide forms on the anode. The thickness of this layer depends on the applied potential difference and on the time for which the current is passed. This oxide film is used as the dielectric in electrolytic capacitors. It may be very thin, less than 10^{-7}m but it has very high insulation strength of some 10^9 V m^{-1}

Breakdown potential for a capacitor

Every capacitor has a working voltage, this is the maximum potential that should be applied between the plates. Any more than this and the dielectric material between the plates will break down and become conducting and the capacitor will be destroyed, usually resulting in a small bang as the material breaks down. Warning: Some badly made power supplies have a capacitor connected across their outputs and so remain live even after the power supply has been switched off. Always be careful when handling apparatus containing capacitors.

Capacitor networks



In practical circuits capacitors are often joined together. We will consider the cases of two capacitors, first in parallel and then in series.

(a) If two capacitors are connected in *parallel*.

The potential across both capacitors is the same (V) and let the charges on the capacitors be Q_1 and Q_2 respectively.

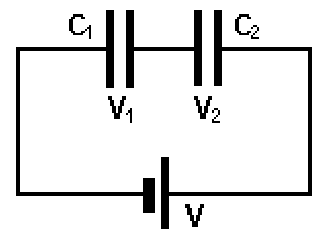
Now Q = CV, and so $Q = C_1V$ and $Q = C_2V$. But the total charge stored $Q = Q_1 + Q_2$, therefore

$Q = Q_1 + Q_2 = V(C_1 + C_2)$

Giving:

Capacitors in parallel: $C = C_1 + C_2$

(b) If two capacitors are connected in *series*.



The charge stored by each capacitor is the same. If V_1 and V_2 are the potentials across C_1 and C_2 respectively then:
$V_1 = Q/C_1$ and $V_2 = Q/C_2$. Therefore: $V = V_1 + V_2 = Q(1/C_1 + 1/C_2)$

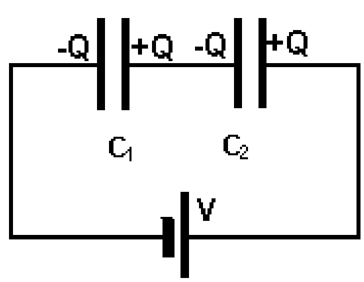
.

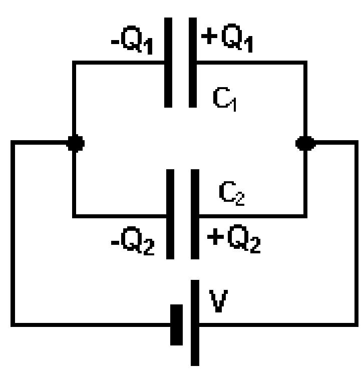
Capacitors in series: $1/C = 1/C_1 + 1/C_2$

(Notice that they are the "reverse" of the formulae for two resistors in series and parallel)

The charge distribution on series and parallel capacitors

When two capacitors are joined tighter in a circuit and then connected to a voltage supply charge will move onto the plates. The actual distribution of charge for a series and parallel circuit is shown below:





The charge and discharge of a capacitor

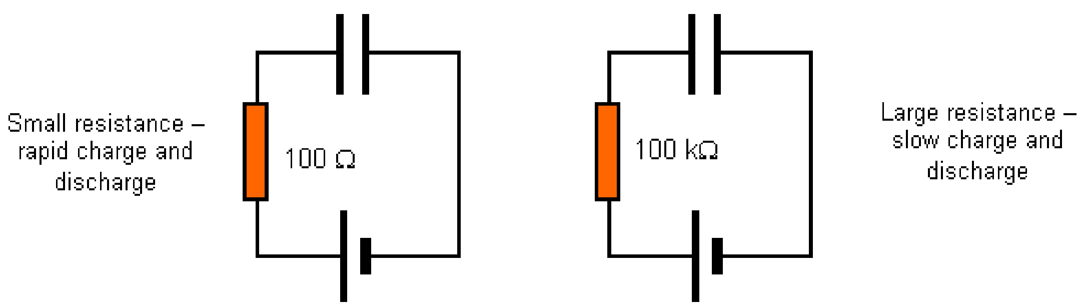
When a voltage is placed across the capacitor the potential cannot rise to the applied value instantaneously. As the charge on the terminals builds up to its final value it tends to repel the addition of further charge.

The rate at which a capacitor can be charged or discharged depends on:
(a) the capacitance of the capacitor and
(b) the resistance of the circuit through which it is being charged or is discharging.

This fact makes the capacitor a very useful if not vital component in the timing circuits of many devices from clocks to computers.

During charging electrons flow from the negative terminal of the power supply to one plate of the capacitor and from the other plate to the positive terminal of the power supply.

When the switch is closed, and charging starts, the rate of flow of charge is large (i.e. a big current) and this decreases as time goes by and the plates become more charged so "resisting" any further charging. You should realise that the addition of a resistor in the circuit in series with the capacitor **ONLY** affects the **TIME** it takes for the capacitor to become fully charge and **NOT** the **EVENTUAL POTENTIAL DIFFERENCE ACROSS IT** – this is always the same and equal to the potential difference across the supply.



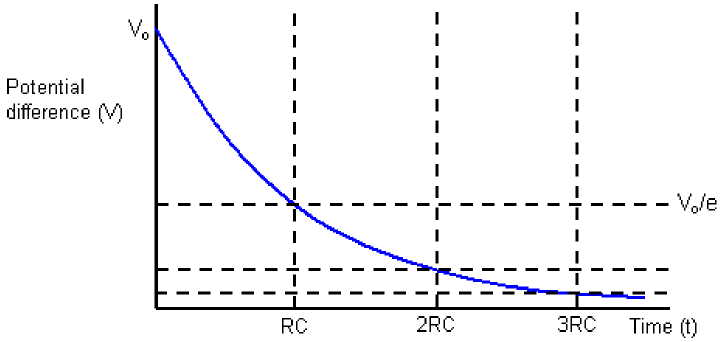
The time constant

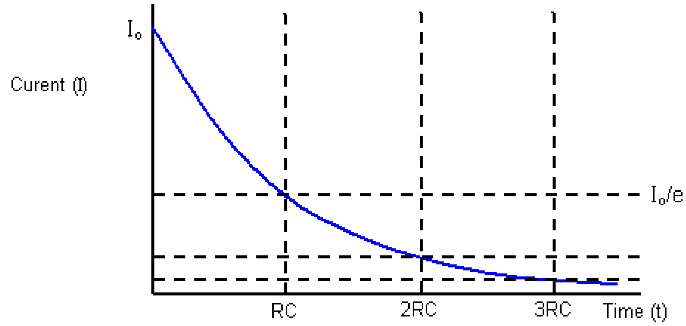
The time that it takes the potential difference across the capacitor to fall to 1/e (37%) of its original value is called the time constant for the circuit. If a capacitor C is discharged through a resistance R then the time constant is equal to RC. You can see that the time constant is independent of the initial voltage and this makes it a very useful quantity when using capacitors in timing circuits.

Time constant (τ) for a capacitor C connected to a resistor R = RC

The voltage falls to 37% of the original value in RC seconds and falls to 14% (37% of 37%) in 2RC s.

Discharging a capacitor

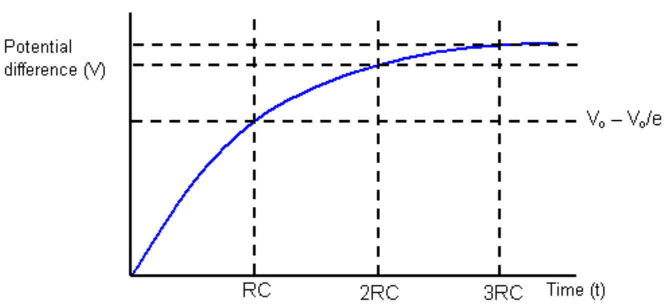


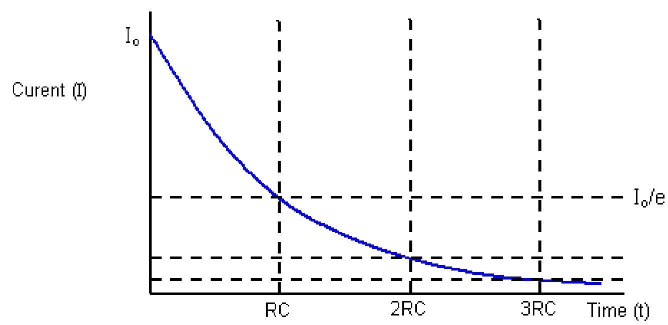


When t = RC, V = V_o/e = 0.37 V_o and the product RC is known as the **time constant** for the circuit. The bigger the value of RC the slower the rate at which the capacitor discharges.

The value of C can be found from this discharge curve if R is known.

Charging a capacitor





As the capacitor charges the charging current decreases since the potential across the resistance decreases as the potential across the capacitor increases.

The area below the current-time curve in both charging and discharging represents the total charge held by the capacitor.

The energy stored in capacitor

The energy stored in a capacitor is given by the equation:

$$E = \tfrac{1}{2}QV$$

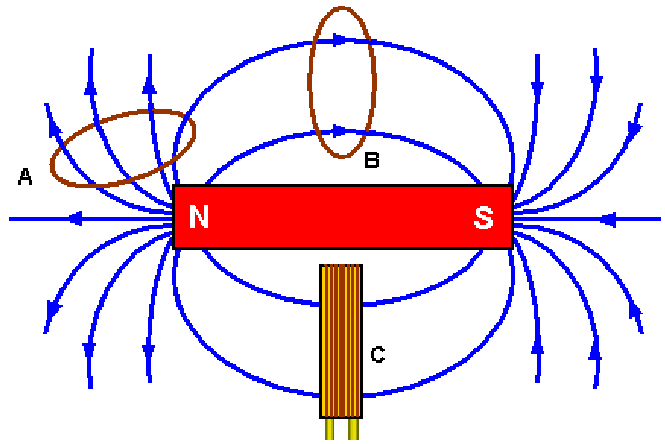
Since the energy supplied by a power source is E = QV this means that capacitors store half the energy supplied to them (the rest is lost as heat in the wires and the power source).

Basic Magnetism

Magnetic fields have a large number of uses in the modern world in, for instance, particle accelerators, plasma bottles, lifting magnets, linear induction motors, hard drive heads and many other applications. Knowledge of those fields has also helped in the studies of the Physics of the van Allen radiation belts, quasars and aurorae.

Flux and flux density

To understand the meaning of magnetic flux (Φ) and magnetic flux density (B) think first about an ordinary bar magnet.



Around the magnet there is a magnetic field and this gives a **'flow of magnetic energy'** around the magnet. It is this flow of energy that we call **magnetic flux (Φ)**. Magnetic flux flows from the north pole of a magnet round to its south pole as shown by the arrows on the lines in the diagram.

Magnetic flux is given the symbol Φ and is measured in units called Webers (Wb).

The amount of magnetic flux flowing through a given area will change from one point to another around the magnet.

In position B there are a smaller number of magnetic field lines passing through the loop than there is when it is in position A so the magnetic field is stronger at A. The amount of flux passing through a unit area at right angles to the magnetic field lines the **flux density (B)** at that point.

Flux density is measured in Tesla (T) where 1 T = 1 Wbm^{-2}

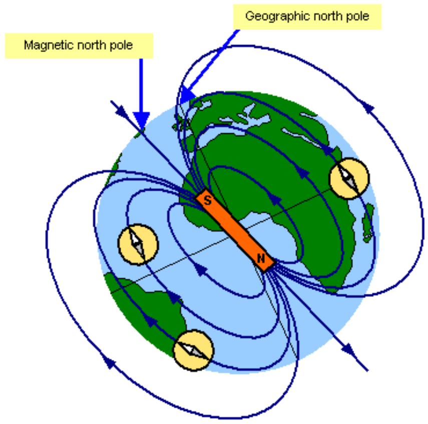
Flux (Φ) = Flux density (B) x area through which flux passes (A) $\Phi = BA$

If we now use more than one loop of wire, in others words a coil of N turns as shown in position C the flux flowing through the N turns is simply N times that flowing through the single loop. The quantity N Φ is called the **flux linkage** for the coil at that point.

Flux linkage = $N\Phi = NBA$

The magnetic field of the Earth

The Earth's magnetic field closely resembles that of a uniformly magnetised sphere, or at least one with a magnetic dipole at its centre.



The Earth's magnetic north pole is not at the same place as the Earth's geographic north pole. At the moment it is to the west of geographic north and moving east but a compass needle will not point to geographic north in London again until around the beginning of the twenty second century. The first records of the position of magnetic north were made in 1659 when it was 11^{o} 15' east. It then moved westwards to be a maximum of 24^{o} 30' west of north in 1820.

The field is not constant with time; it changes over periods as short as a few hundred years. It is thought that it is due to the motion of molten material within the Earth's core. The field has also undergone periods of reversal, the direction changing by 180^{o}. The reasons for this are not too well understood but a study of the magnetisation of rocks, a science known as paleomagnetism, has been helped our understanding of continental drift.

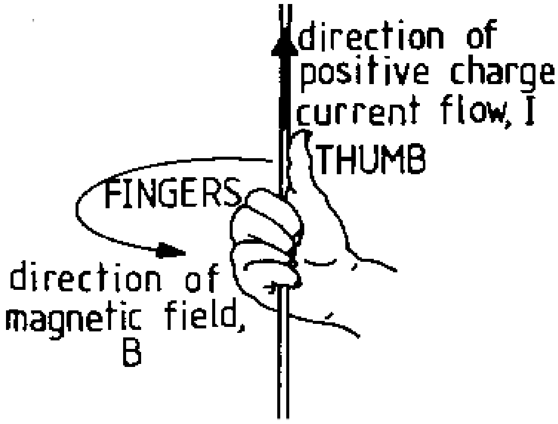
At any point on the Earth the resultant magnetic field may be considered in two components:
(a) the vertical component and (b) the horizontal component.

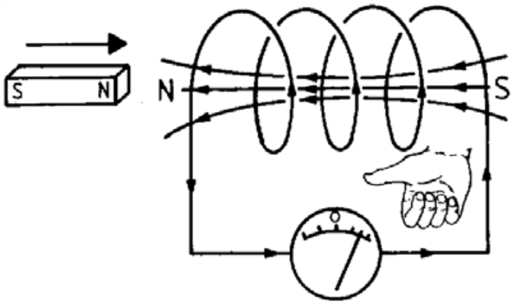
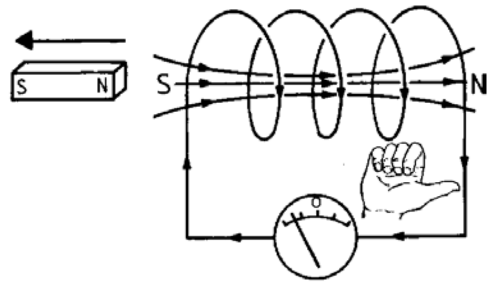
The direction of the resultant field makes an angle ø with the horizontal, and this angle is known as the angle of dip. This is related to the two components by the formula:

$$\tan \phi = \text{vertical component/horizontal component}$$

Electromagnetic Induction: Electromagnets (current makes magnetism)

A current carrying wire produces a magnetic field. The diagram below shows the direction of the magnetic field when using conventional current.



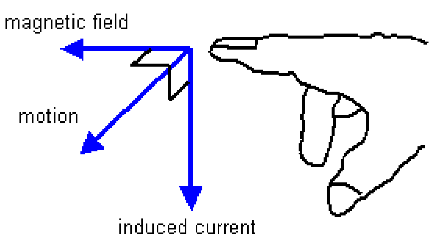
When an electric current flows in an anti-clockwise direction in a current- carrying coil, the end of the coil behaves likes the North pole of a magnet as predicted by:	When an electric current flows in an clockwise direction in a current- carrying coil, the end of the coil behaves likes the South pole of a magnet as predicted by:
	
	

Electromagnetic Induction: Induction (magnetism makes current)

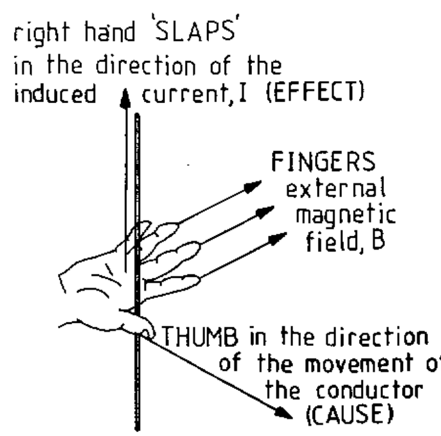
Fleming's right-hand rule

Fleming proposed a simple rule for giving the direction of the induced current to explain Faraday's experiments:

If the thumb and first two fingers of the right hand are held at right angles and the first finger is pointed in the direction of the magnetic field and the thumb in the direction of motion then the second finger gives the direction of the induced current



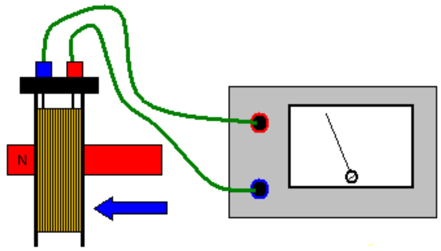
The right hand slap rule shows the direction of the induced current where the thumbs indicates the direction of the movement and the fingers pointing at 90^{o} point in the direction of the magnetic field.



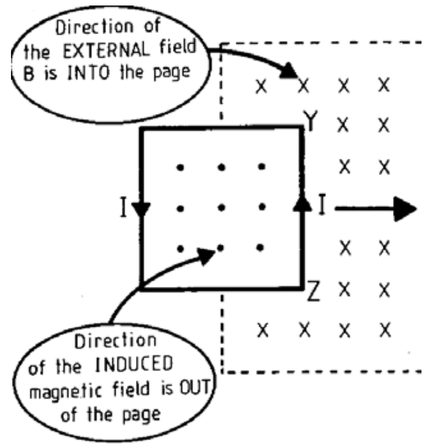
Electromagnetic induction is used in most modern devices.

The ignition and sparking system in many cars uses an induction coil
Transformers are used in thousands of different appliances
Electricity is generated in power stations using generators
Electricity is transmitted round the country at very high voltage
Electric guitars are ways of making music
Clockwork radios can pick up signals in remote places

Electromagnetic induction may sound rather complicated but all it means is a way of generating electricity by using moving wires, moving magnets or changing the voltages in one coil to make electrical energy in another.

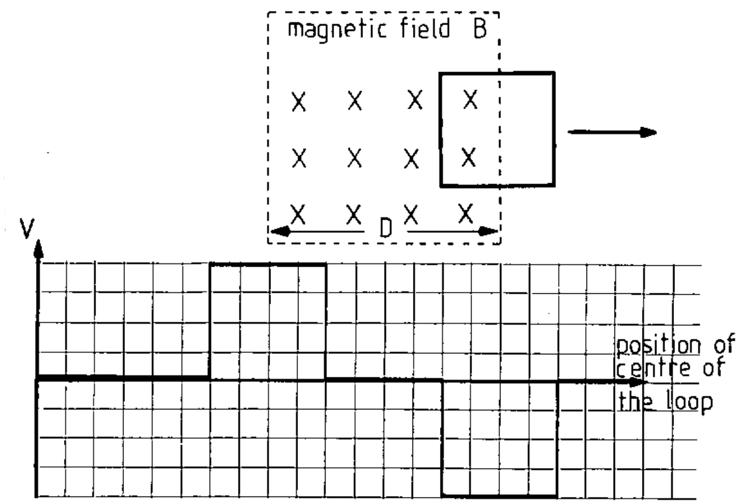


A voltage (E.M.F) can be induced in a wire or a coil if the wire is in a region where the magnetic field is changing. This can be done by:
(a) moving the wire through a fixed field
(b) moving a fixed field (a permanent magnet) relative to the wire or
(c) varying the field by using A.C. in a coil

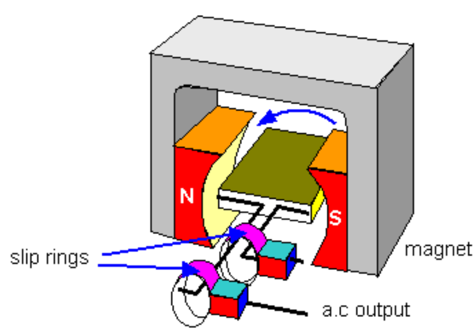


When a coil is moved into a magnetic field, the current induced in the wire can be predicted by the right hand slap rule.

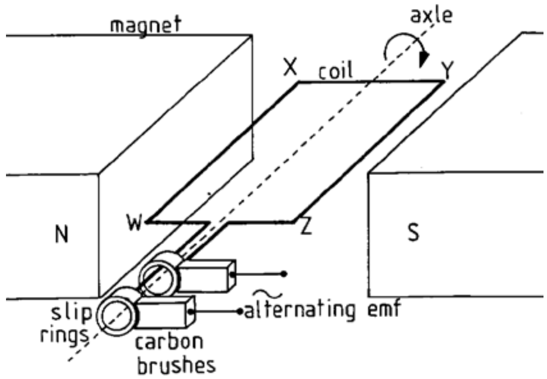
Current cannot be made when the coil is completely in or out of the magnet field as the two currents cancel each other out (the current flowing around the coil is a consequence only of the current induced between Z and Y).
The graph below shows the induced current as a coil passes through a magnet field.



However, you can create a resultant current in a constant magnetic field if you rotate the wires.

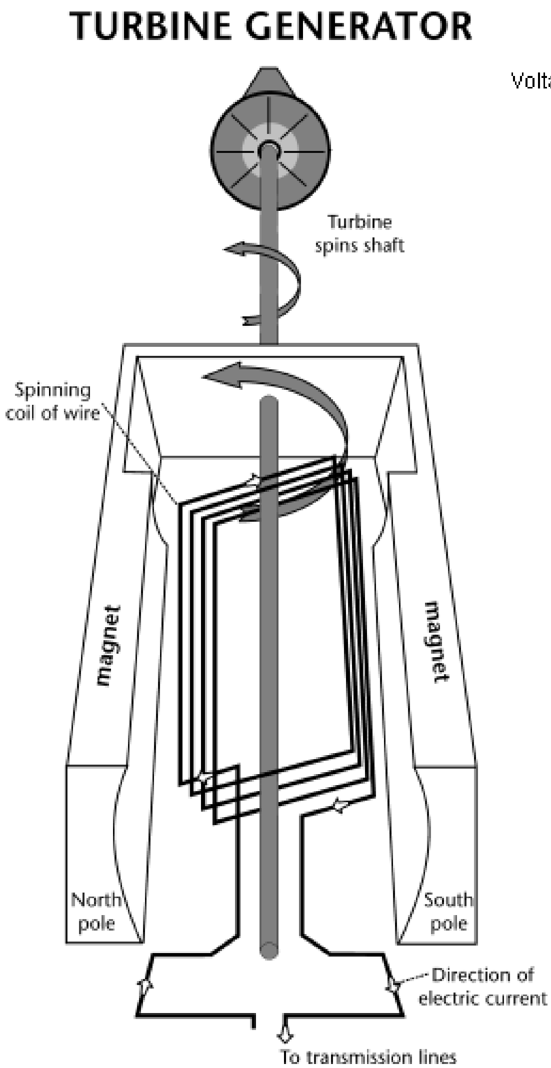


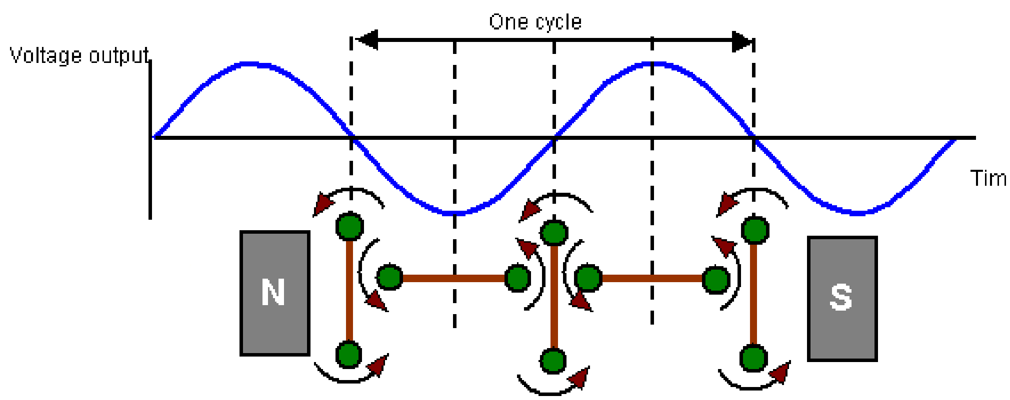
This effect is used in electrical generators (the one shown is an A.C. generator).



The right hand slap rule can be used to predict the current flowing from Y to Z as wire ZY is moving downwards relative to the magnetic field as it spins.
The right hand slap rule can be used to predict the current flowing from X to W as wire WX is moving upwards relative to the magnetic field as it spins.

A current is produced in a clockwise direction around the coil when it is momentarily in this position.





Over a complete rotation the current changes sinusoidally.

When the lengths ZY and WX are moving parallel to the magnetic flux, no current is induced.
Maximum current is induced when the ZY and WX are moving perpendicular to the flux lines.
ZY and WX are always moving in opposite directions so the induced currents are never cancelled.
Slip rings are used – otherwise the wires would twist more on each rotation.

All generated electricity in New Zealand is made this way.
In commercial generators the performance of both types is improved by having:
(a) a radial magnetic field produced by electromagnets with mutiple coils and
(b) a large number of coils on the rotor

As the coil rotates it cuts through the lines of magnetic flux producing an induced E.M.F

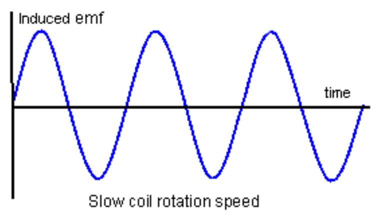
In generators where the output current may be very large, as in a power station, it is the magnet that rotates while the coil remains at rest.

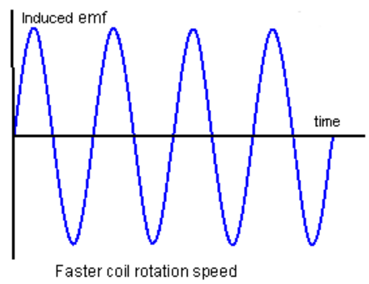
In modern alternators installed in a power station the E.M.F generated will be some 25 kV and the current produced over 1000 A!

Effect of changing the speed of rotation of the coil on the induced E.M.F. If the speed of rotation of the coil is changed two things happen

(a) since the rate of cutting of magnetic flux is increased the output E.M.F will be increased also in line with Faraday's law

(b) the frequency of the output E.M.F will be increased as well since the coil makes a revolution in a shorter time





Induced E.M.F

When the magnetic flux through a coil changes, the E.M.F generated in the coil can be expressed as:

$$\varepsilon = -\frac{\Delta \phi}{\Delta t}$$

This change of flux ø can be produced by either:
(a) moving the wire or coil through the field or
(b) changing the intensity of the magnetic field

If we think of a conductor moving through a constant magnetic field then the E.M.F (E) generated between the two ends of the conductor at any moment is given by the equation:

$$E = -\frac{N d\Phi}{dt}$$

where N is the number of conductors cutting the flux (If there is only one wire cutting the field N = 1).

For an electrical generator this can be made into the equation:

Maximum e.m.f: $E_0 = BAN\omega$

Lenz's law

The direction of the induced E.M.F was explained by Lenz who proposed the following law in 1835:

> The direction of the induced e.m.f. is such that it tends **to oppose** the change that produced it.

We can explain this law by considering the energy changes that occur when a magnet is moved towards a coil. Assume that the magnet is moved towards the coil with its north pole facing towards the coil. Now by Lenz's law this should induce a current in the coil such that the right-hand end of the coil (B) nearest the magnet is also a north pole. If this is true then it should repel the magnet and work must be done on the magnet to move it in against this repulsion.

The energy used goes to produce the induced E.M.F in the coil.

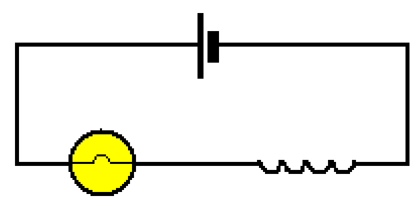
Since the E.M.F generated opposes the changes that produce it, it is known as a **back E.M.F.** This effect is particularly important in electric motors.

Self-inductance

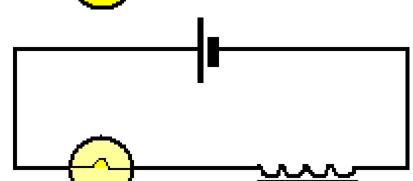
If the current through a coil is altered then the flux through that coil also changes, and this will induce an E.M.F in the coil itself. This effect is known self-induction and the property of the coil is the self-inductance (L) of the coil, usually abbreviated as the inductance.

$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

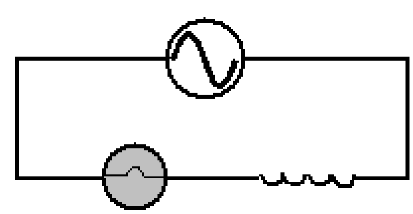
An air-cored inductor is connected in series with a d.c. supply and a 12 V bulb.

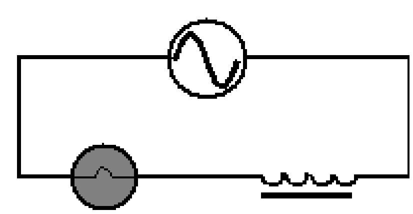


The resistance of the solenoid will be low so that it barely affects the light emitted by the bulb, and placing an iron core inside the inductor will make no difference to the bulb's brightness.



If the experiment is repeated using a.c. with an air core, the inductance will probably prevent the lamp from reaching its full brightness. If an iron core is placed inside the solenoid, however, its inductance is increased considerably and the lamp goes out due to the increased self-inductance and resulting back E.M.F in the coil. The coil and iron rod are called a choke.





A single wire connected to a cell and doubled back on itself has no net magnetic field - the field produced by the current in one direction cancels that produced by the current in the other. This is known as non-inductive winding.

Energy stored in an inductor

Since a changing current in an inductor causes an E.M.F if the source supplying the current is to maintain a p.d. between its terminals the inductor must gain energy.

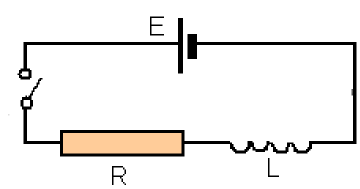
Energy stored in inductor = ½ LI^2

The energy is used to produce the magnetic field in and around the coil. If the current is suddenly interrupted a spark may occur as the energy is dissipated. Self- inductance can be a problem in circuits, where the breaking of the circuit can induce a large E.M.F, and so the switches maybe immersed in oil to quench the arc. Alternatively a capacitor may be connected across the terminals to slow down the decay of current and so reduce the induced E.M.F

The solenoid plays a rather similar role with relation to magnetic fields as the capacitor does to electric fields - the ability to store energy.

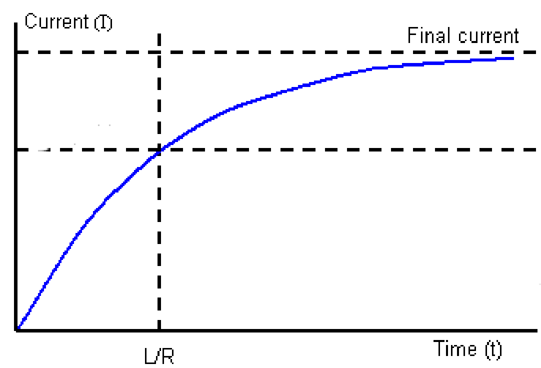
Growth and decay of current in an inductor

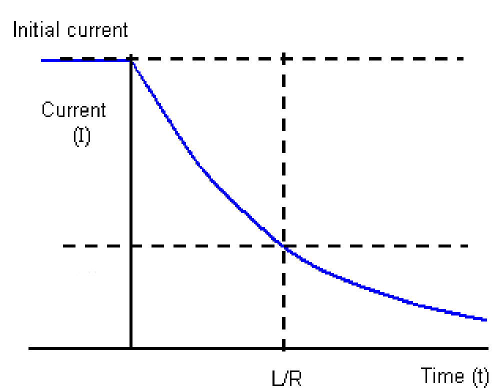
When a battery of e.m.f E is connected across a resistor and an inductor in series the current does not rise to its final value instantaneously. There is a rise time that is due to the back e.m.f (ε) in the inductor and the resistance and inductance of the circuit.



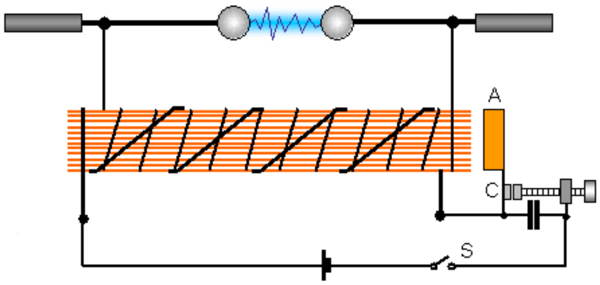
The time constant (time for 63% change) for an inductor in a DC circuit is:

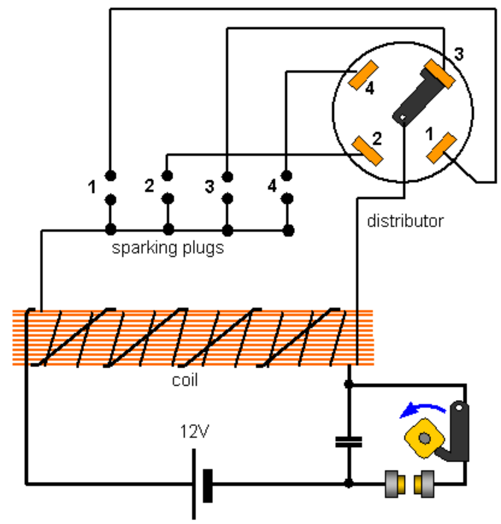
$$\tau = \frac{L}{R}$$





The induction coil





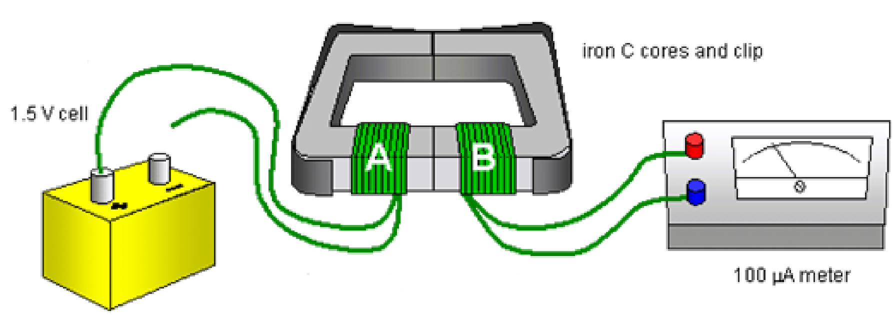
Invented in 1851, the induction coil is a piece of apparatus used to produce very high voltages from a low voltage D.C. supply.

When the switch (S) is closed a current flows through the primary coil this magnetises the soft iron rods in the core of the coil. These then attract the soft iron armature at A which therefore breaks the contact at C. The primary current falls to zero very quickly. This sudden drop in current induces a large voltage in the secondary coil that has a large number of turns. As soon as the contact is broken the magnetic field of the rods disappears, the armature springs back and makes the contact at C once again. The process then repeats itself. Voltages of many tens of thousands of volts can be made from a 6- 12 V input.

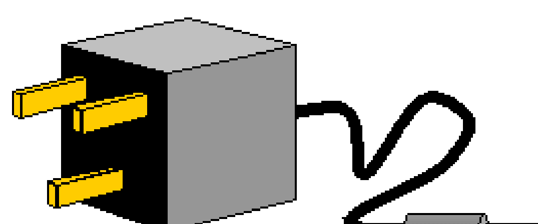
Induction coil form the ignition circuit of a modern petrol-driven cars.

The transformer

In the diagram below, if the wire is connected to the battery, a current will flow in coil A. This will be like bringing up a magnet to coil B and so a current will flow in B. If the switch is held fixed to the battery the current will fall to zero, but if the wire is disconnected a current will flow in B in the opposite direction but will stop after the switch has been opened. Connecting quickly will give a larger current than if the wire is slowly pressed against the battery terminals.

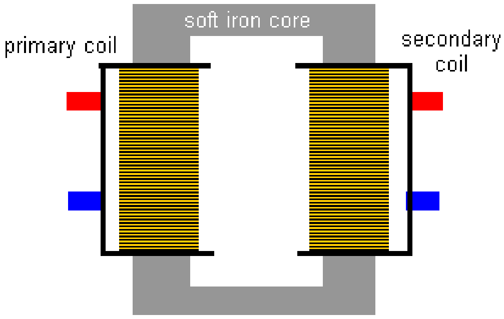


Note: strictly speaking in all these experiments it is a voltage that is generated and this then gives a current in the meter if the circuit is complete.



If we wish to run a 12 V toothbrush from the 240 V mains or change 20 000 V to 400 000 V for transmission in a power cable then we must use a transformer.

A transformer consists of two coils - the primary coil and the secondary coil both wound on a soft iron core.



If an ac. voltage is applied to the primary coil then this will produce a changing magnetic field in the iron core. This then induces an alternating voltage in the secondary coil.

Remember that you will only get a voltage in the secondary coil if the magnetic field is changing. Therefore:

Transformers will only work with ac.

The number of turns on both the primary and secondary coils affects the output voltage of the transformer. In fact the ratio of the output voltage to the input voltage is the same as the ratio of the number of turns on the secondary to the number of turns on the primary.

$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$

If the number of turns on the primary is greater than that on the secondary then the voltage on the secondary will be less than that on the primary If there are more turns on the secondary the output voltage will be larger.

If the secondary voltage is less than the primary voltage, the transformer is known as a STEP-DOWN transformer. If the secondary voltage is greater than the primary voltage it is known as a STEP-UP transformer

A voltage change from primary to secondary will mean a current change also. If the voltage is increased the current will be decreased and vice versa.

So a 20:1 step-down transformer for voltage will be a 20:1 step-up transformer for current. Bigger currents need thicker wire and so step down transformers have primary coils of thin wire and secondary coils of thick wire. Examples of step up and step down transformers:

(a) Step down: electric mains clock, stereo, substation, low voltage power supplies, and audio systems in televisions.

(b) Step up: power station end of transmission cables, electron gun in a TV, "starter" coils in fluorescent lights.

Energy losses in a transformer

Transformers are almost 100% efficient. However, some energy is always lost and so the output voltage will be a little smaller than the calculated value.

Energy can be lost as:

(a) Heat in the coils because of the resistance of the wire;
(b) Incomplete transfer of magnetic field;
(c) Heating of the core due to induced currents in it. This is reduced by making the core out of insulated soft iron in laminated strips. If this were not done the cores of large transformers would get so hot that they would melt.

The transmission of electricity

Electricity is generated in power stations and from there it is transmitted across the country by power cables to towns and villages. As you know, all cables have resistance and so as the electricity passes through them it will lose energy as heat in the cable. The energy lost in a power cable that is carrying a current I and is of resistance R is given by the formula:

$$\text{Power loss (W)} = I^2R$$

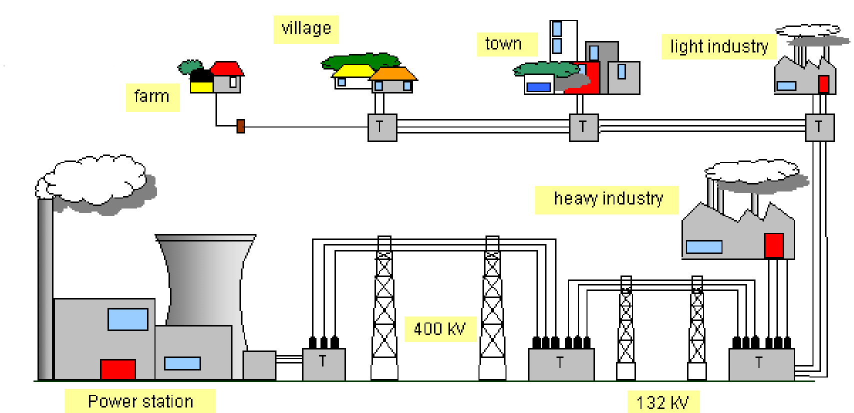
The power loss depends on the resistance of the cable and so to reduce this power loss the resistance of the cable must be as small as possible.

So changing the resistance is not a practical idea, but the current passing through the cable can be altered using a transformer. If electricity is transmitted at low currents and high voltages, much less electricity is lost.

To keep the power loss as small as possible the transmitted voltage is very high. Electricity generated in a power station at 25 000 V (25 kV) is stepped up to 275 kV or 400 kV for transmission across large distances. Near towns, villages and industrial sites there are transformers that step down the voltage ready for use

The grid system

The diagram shows how electricity is transmitted round the country by the national grid system. Note the step-up and step-down transformers and the voltages used.



Alternating voltage and current

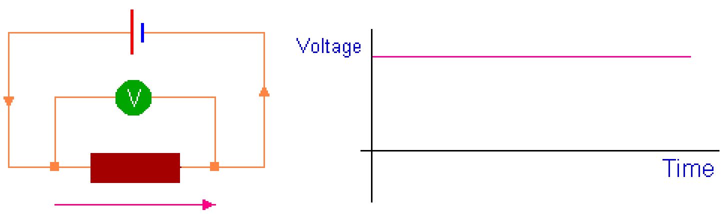
Although the wiring to the rotating loop in an electrical generator can be arranged, using a commutator, so that the output is pulses of current in one direction, there are practical advantages in distributing electric power by an AC system rather than a DC one.

These include:

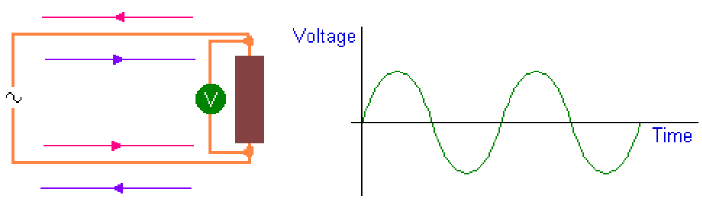
- For electronic appliances which require DC, conversion from AC is simple using a rectifier.
- Electric power for heating, lighting and motion can be obtained just as readily using AC as DC. In fact the same relationships between power, current, voltage and resistance that are used for DC can be used for simple AC circuits provided the AC values of these quantities are defined and used correctly.

AC and DC circuits

A source of energy which provides charge flowing in only one direction is a DC supply. DC means direct current.

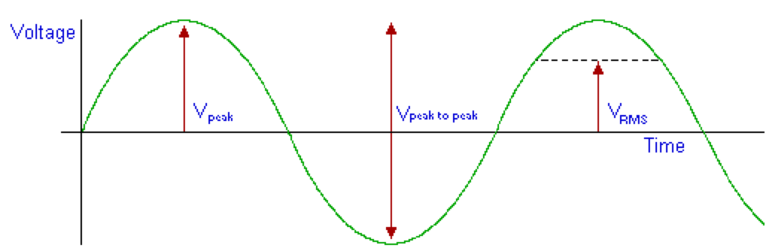


When charge moves in one direction then reverses direction, the current direction changes from a maximum positive flow, through zero, to maximum negative flow. The source of energy causing the charge to move then change direction is an AC supply. AC means alternating current.



The number of complete direction changes per second is the frequency, measured in hertz. The frequency is the inverse of the period. f = 1/T where f = frequency (Hz)
The AC voltage can be described using peak voltage (V_p), peak-to- peak voltage (V_{pp}), or more commonly, rms voltage (V_{rms}).

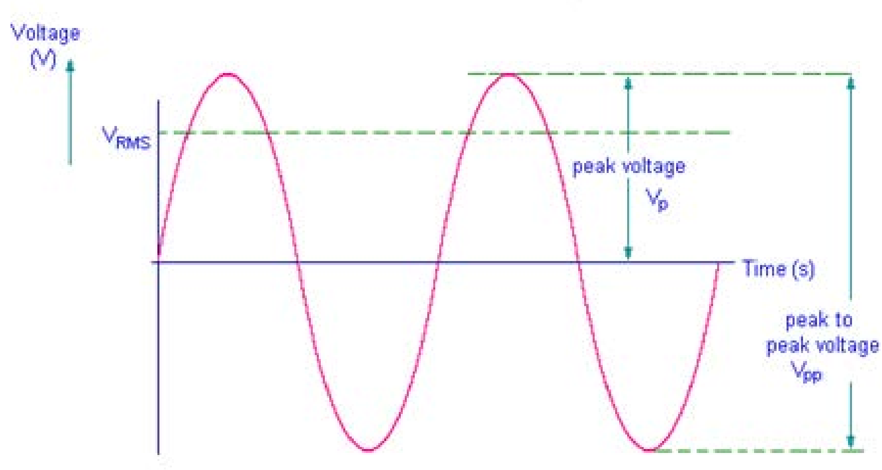
A value for AC voltage given as rms voltage is the equivalent of the DC voltage which would deliver the same power to the component. For example, 10 V DC put across a light will make it glow brightly. When 10 V **peak** AC is put across the light, it is not as bright as before because the AC is not continuously at 10 V, but increasing and decreasing.



From 0 V the voltage rises to +10 V then drops through 0 V to -10 V, before rising again to 0 V 50 times each second.

In an AC circuit, Ohm's law (V =IR and related formulas) can still be applied using either peak values or rms values.

AC Values



Peak values of voltage, V_p (and I_p), are maximum values.
Peak to peak values, V_{pp} (and I_{pp}), are the differences between minimum and maximum values.

It is convenient to have a type of average value of current and voltage rather than deal with the continuously altering instantaneous values.

A simple average of all the instantaneous values would give 0 (equal number of positive and negative values), but the Root Mean Square technique gives a value which can be used as the equivalent of a DC current or voltage in electric power calculations. The rms values of V (and I) are related in a simple way to peak values.

$$I_{MAX} = \sqrt{2}\, I_{rms} \qquad V_{MAX} = \sqrt{2}\, V_{rms}$$

Where

V_{rms} = root mean square voltage and V_{MAX} is the peak voltage
I_{rms} = root mean square current and I_{MAX} is the peak current

The mains power supplied to houses in New Zealand is described as 240 V, 50 Hz.

This voltage is the rms value and hence the peak voltage from domestic mains wiring is:
$V_{MAX} = \sqrt{2} \times 240 = 339$ volt

The 50 Hz figure refers to the frequency of the voltage cycle.

Alternating currents

Direct current is an electric current that flows in one direction only – the electrons drifting down the wire towards a definite end. If they change direction, first moving one way and then the other we have what is called an **alternating current**.

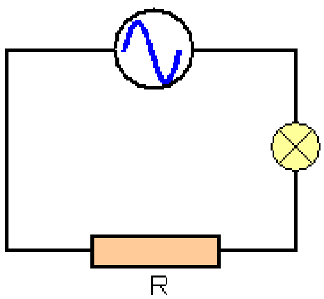
Resistors behave identically in A.C. and D.C. circuits – they are **resistive.**

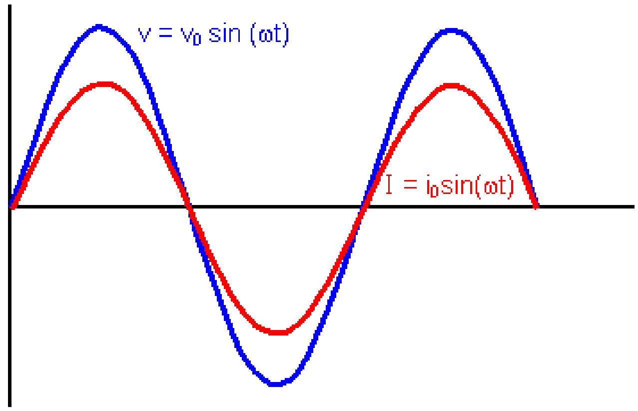
Capacitors and inductors behave differently in A.C. and D.C. circuits – they are **reactive.**

Resistance

In a circuit containing only resistance the current and voltage are in phase, and the equations for their variation with time are:

$$I = I_{MAX} \sin \omega t \qquad V = V_{MAX} \sin \omega t$$





Because I and V are in phase, Ohms law can still be applied at any instant and R will be the same irrespective of the frequency of the supply. Resistance is measured in ohms.

Remember:

$$V = IR$$

Reactance

The term reactance is given to the effective resistance of a component to A.C. It is given the symbol X and is defined as the amplitude of the voltage across a component/amplitude of the current flowing through it

For a **capacitor**

$$X_C = \frac{1}{\omega C}$$

where

$$\omega = 2\pi f$$

The reactance of a capacitor is therefore inversely proportional to the frequency of the applied p.d. Reactance is measured in ohms.

For an **inductor**

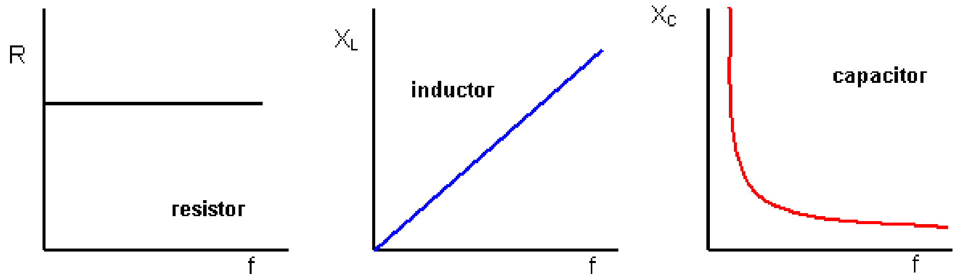
$$X_L = \omega L$$

where

$$\omega = 2\pi f$$

The reactance of an inductor is therefore directly proportional to the frequency of the applied p.d. Reactance is measured in ohms.

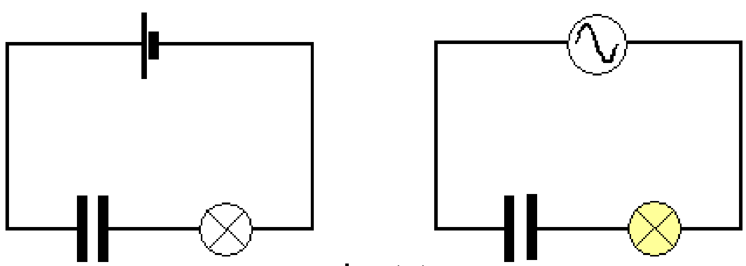
The variation of the resistance of a resistor and the reactance of an inductor and a capacitor with the applied voltage frequency (f) is shown below:

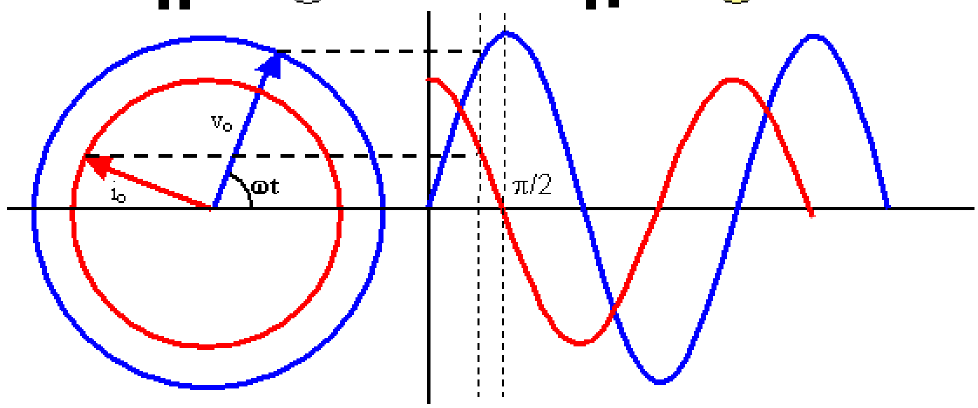


Capacitative circuits

When a capacitor is connected to a voltage supply the result is quite different for an A.C. supply from that for D.C.
In a D.C. circuit no current flows after a while and the lamp does not stay light. But if the supply is A.C. the lamp lights showing that some current must be flowing through it. This can be explained as follows.

When a capacitor is connected to an A.C. supply the plates of the capacitor are continually charging and discharging, and so an alternating current flows in the connecting wires. Current does not actually flow through the capacitor itself. Initially there is no voltage across the capacitor. As soon as the voltage begins to increase a large current flows. This current falls as the capacitor charges up, i.e as the p.d across it increases, and is zero when the capacitor is fully charged.

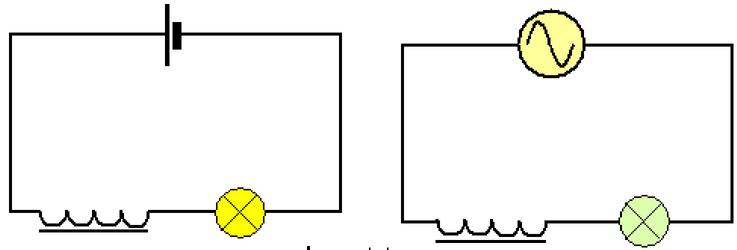




The current and voltage are not in phase; in fact, the current is 90^o ahead of the voltage, as shown by the phasor diagram.

In a capacitative circuit the current leads the voltage by 90^o (π/2)

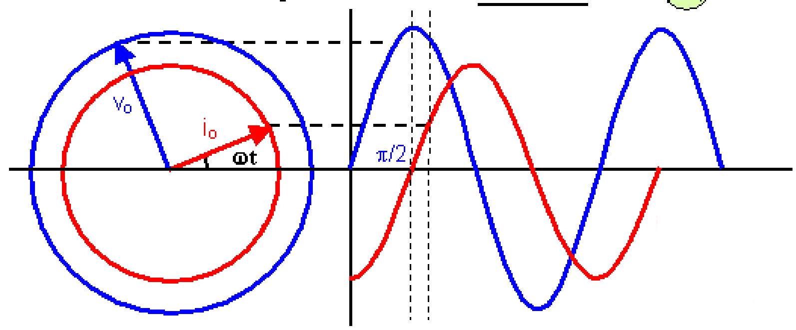
Inductive circuits



Like a capacitative circuit, a pure inductive circuit behaves quite differently from one containing resistance alone.

When an inductor is connected to a voltage supply the result is quite different for an A.C. supply from that for D.C.

In a D.C. circuit the lamp may be slower to light when it turns on but then stays lit. But if the supply is A.C. the lamp stops glowing particularly if an inductor with iron core is used



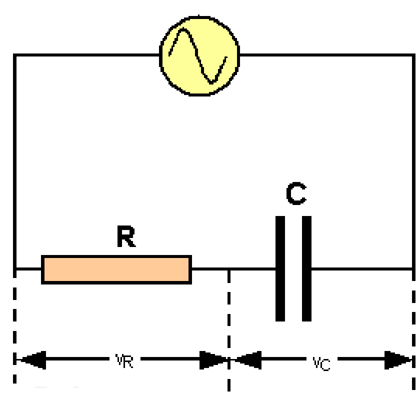
If D.C. were used, however, the lamp would remain alight whether there was an iron core or not.

In the inductive circuit the current and voltage are not in phase; in fact, the current lags behind the voltage by 90^o as shown by the phasor diagram.

In inductive circuits the voltage leads the current by 90°.

When these components are combined in A.C. circuits, interesting and useful things happen**.**

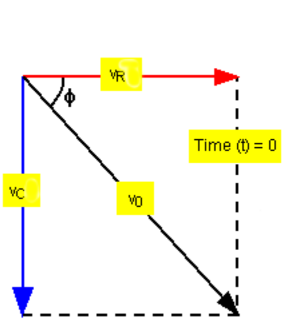
The CR (capacitance-resistance) circuit

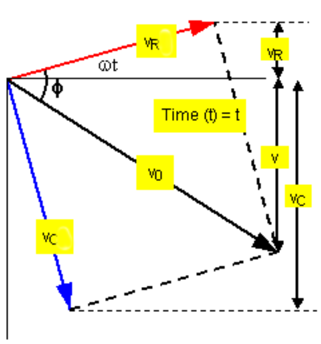


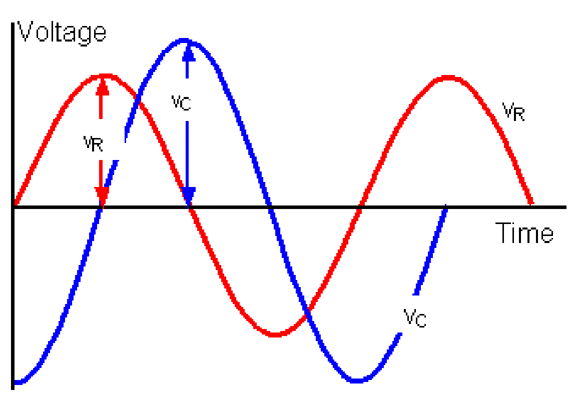
The A.C. circuit shown contains both resistance and capacitance, and therefore both the component and the frequency of the supply voltage affect the current in the circuit.

The A.C. resistance of such a circuit is known as the impedance of the circuit and is denoted by the symbol Z. Impedance is measured in ohms.

We will now deduce the impedance of the circuit using the vector treatment. Consider the voltages round the circuit. The supply voltage will be denoted by Vo and the voltages across the resistor and capacitor by V_R and V_C respectively. We know that for a resistor the current and voltage are in phase, while for a capacitor the current leads the voltage by 90^o; V_R therefore leads V_C by 90^o, as shown in the phasor diagram at t = 0 and t = t.



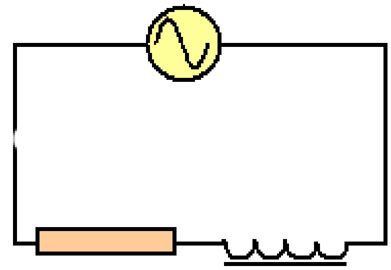




The impedance Z is calculated by:

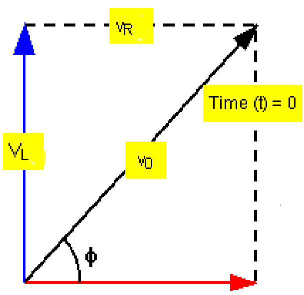
$$V = IZ$$

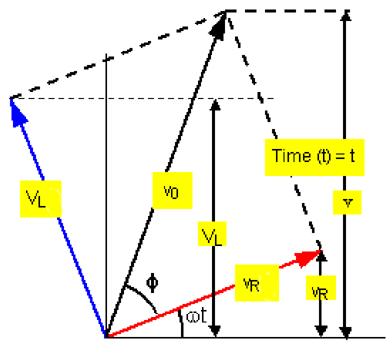
The LR (inductance-resistance) circuit

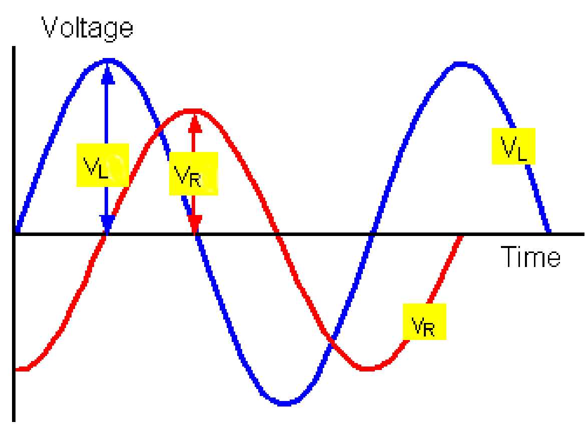


The circuit below contains both inductance and resistance. As with the capacitance-resistance circuit, the current through it depends on the value of both the components and the frequency of the supply voltage.

Let the supply voltage be V_0 and the voltages across the inductor and the resistor be V_L and V_R respectively. Now we know that for a resistor the current and voltage are in phase, while for an inductor the current leads the voltage by 90^o; V_L therefore leads V_0 by 90^o.



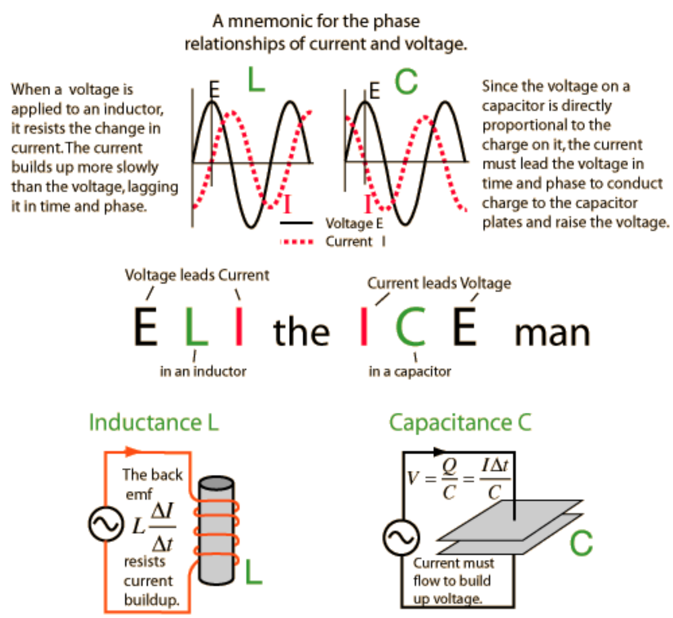




The impedance Z is calculated by:

$$V = IZ$$

ELI the ICE man



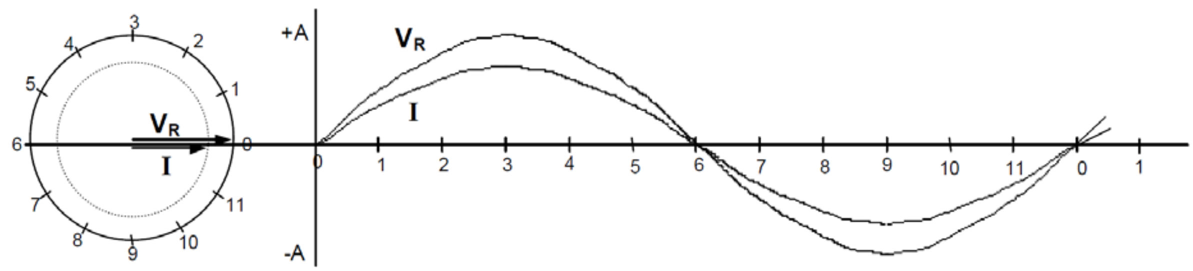
PHASORS

A **phasor** is like a clock hand which follows the point around in an anticlockwise direction.

A point starts at position 0 on the reference circle and moves anti-clockwise with constant angular velocity (ω). The vertical displacement for positions 1, 2, 3, …., are shown by the dotted lines. These displacements can then be transferred to the time or angle graph.

AC circuits

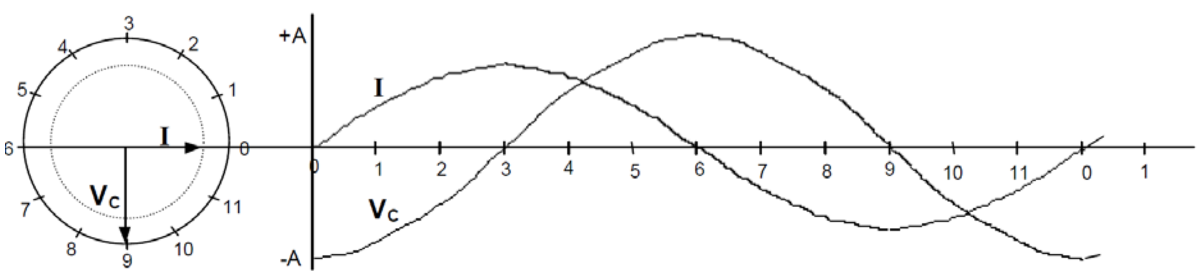
Voltage and current are in phase for a resistor.

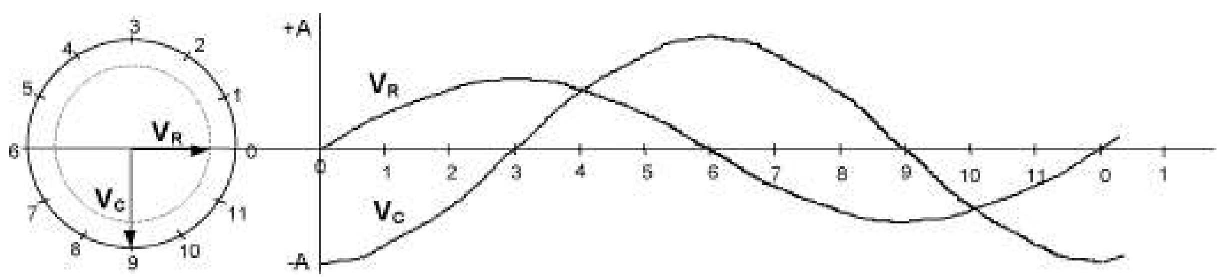


RC circuits

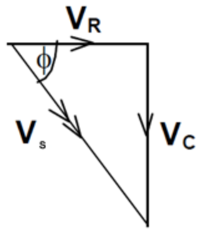
Voltage lags current by ½ π rad. or 90º for a capacitor.

No current can flow when the plates are charged up to maximum voltage. Maximum current will flow when there is no charge on the plates (there is no repulsive force against charge moving onto them).





Voltage for a resistor is in phase with current. Since the generator must supply both the voltage to the resistor (V_R) and the capacitor (V_c), the supply voltage (V_S) must be the resultant of these two phasor vectors.



$$V_s^2 = V_R^2 + V_C^2$$

$$V_s = \sqrt{V_R^2 + V_C^2}$$

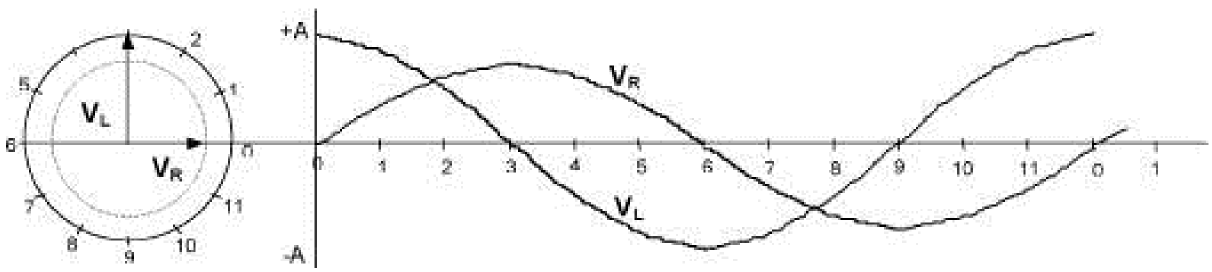
$$\tan\phi = \frac{V_C}{V_R}$$

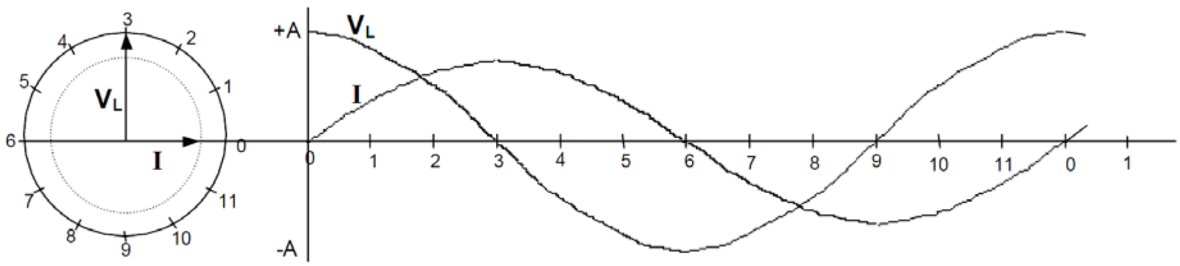
The angle Φ is the phase angle that the supply voltage lags the current.

RL circuits

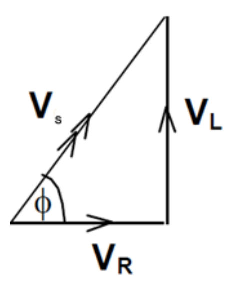
Voltage leads current by ½ π rad. or 90º for an inductor.

When a driving current flows through the first turn of a coil, magnetic lines of force (flux) will spread out through the next turn causing an induced current in the opposite direction to the driving current. This will delay the build up of the driving current, since it will happen with each turn of the coil. The opposite will happen when the driving current reduces.





Voltage for a resistor is in phase with current. Since the generator must supply both the voltage to the resistor (V_R) and the inductor (V_L), the supply voltage (V_S) must be the resultant of these two phasor vectors.



$$V_s^2 = V_R^2 + V_L^2$$

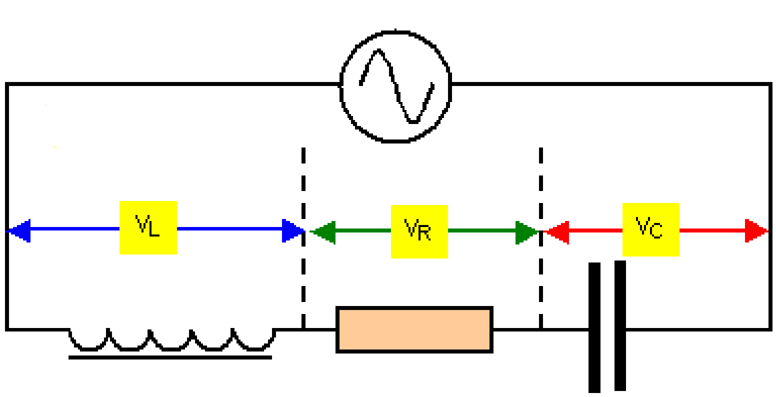
$$V_s = \sqrt{V_R^2 + V_L^2}$$

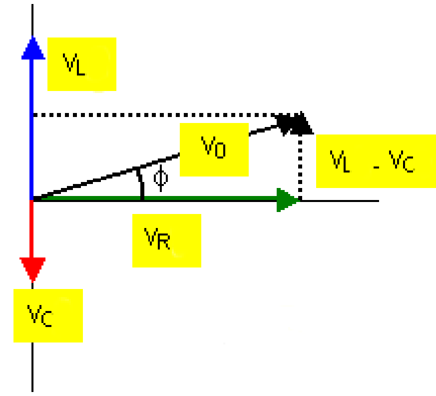
$$\tan\phi = \frac{V_L}{V_R}$$

The angle Φ is the phase angle that the supply voltage leads the current.

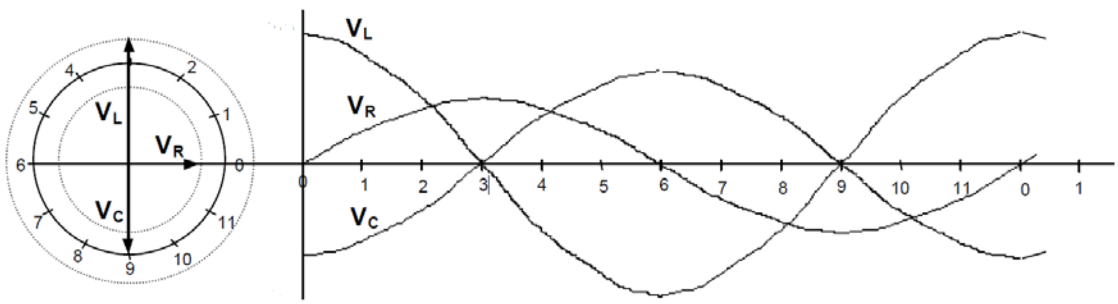
The L-C-R series circuit

Combining an inductor, a capacitor and a resistor in series produces the following:





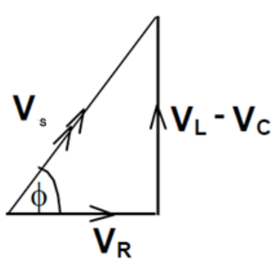
For all 3:



The voltages across the inductor and the capacitor (V_L and V_C) are 180^o out of phase, and the result of the addition of these two must be added vectorially to V_R to give the resultant voltage, which is therefore given by:

$$V_0^2 = (V_L - V_C)^2 + V_R^2$$

Voltage for a resistor is in phase with current. Since the supply voltage must supply both the voltage to the resistor (VR), the capacitor (VC) and the inductor (VL), the supply voltage (VS) must be the resultant of these three phasor vectors.



$$V_s^2 = V_R^2 + (V_L - V_C)^2$$

$$V_s = \sqrt{V_R^2 + (V_L - V_C)^2}$$

The angle *Φ* is the phase angle that the supply voltage leads the current. It is calculated by:

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

(This example assumes $V_L > V_C$)

This means that the impedance of the circuit is:

Impedance of LCR circuit: $Z = [(X_L - X_C)^2 + R^2]^{1/2}$

It should be realised that since the voltages across the capacitor and inductor are 180^o out of phase they may be individually greater than the supply voltage!

Resonance

One very important consequence of this result is that the impedance of a circuit, Z, has a minimum value when $X_L = X_C$.

When this condition holds the current through the circuit is a maximum.
This is known as the resonant condition for the circuit.

Since X_L and X_C are frequency-dependent, the resonant condition depends on the frequency of the applied A.C.
Since $X_L = \omega L$ and $X_C = 1 / \omega C$, the impedance is minimum when $\omega L = 1 / \omega C$.

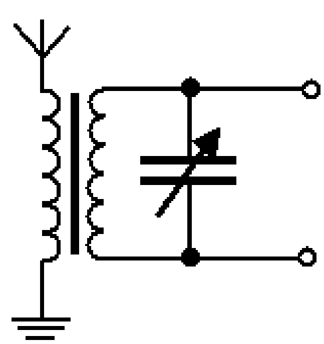
At this point, $f = 1/2\pi\sqrt{LC}$

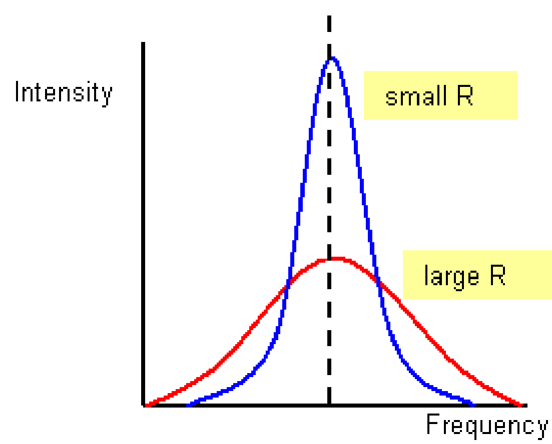
The frequency when this occurs is called the **resonant frequency.** By adjusting L and/or C the circuit can be tuned to a desired resonant frequency.
Every series A.C. circuit has a frequency for which resonance occurs, known as its resonant frequency (f_o).

If the capacitor (or inductor) is variable, then the circuit may be tuned to resonate at a particular frequency. This is used in the tuning of a radio set.

The aerial receives a broad hand of frequencies and the capacitor is varied so that the circuit resonates at the frequency of the required station. A simple circuit for the tuner section of a radio receiver is shown below.

The response of the circuit with frequency is also shown below, in which R is the total series resistance of the tuned circuit.





By using a variable capacitor as a tuner in a radio circuit different stations may be picked up. The large current at resonance being fed to an amplifier and finally to operate the loudspeaker.